HYDRAULIC RESISTANCES AND HEAT EMISSION IN THE STABILIZED FLOW OF NON-NEWTONIAN FLUIDS

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Velocity fields, hydraulic resistances, and heat emission coefficients are considered for the laminar flow of structurally viscous fluids with exponential flow law, extended also to a medium with a significant appearance of elastic properties. Methods of an experimental investigation of the hydrodynamics and heat exchange of non-Newtonian fluids are discussed.

1. Generalized Exponential Flow Law

Mathematical physics models of flowing media are constructed in modern hydrodynamics by means of a set of macroscopic properties. Hence, diverse flow domains, or a different formulation in studying some aspects of the given flow, can be described by different mathematical models, starting from the principle of greatest simplicity in describing the primary aspect of the considered problem. Thus, for a sufficiently rapid unbounded fluid flow around a solid body, the pressure distribution along the body outline is usually described well enough by the Euler ideal fluid model, which possesses just one essential physical property, the density ρ. At the same time, the aerodynamic drag and stream parameter distribution (velocity, temperature, impurity concentration) in the direct neighborhood of the body depend essentially on at least one property of the medium, the dynamic viscosity μ.

A fluid with viscosity independent of the flow kinematics is called Newtonian. An ideal gas is a completely Newtonian fluid since its viscosity depends only on the thermodynamic state parameters and is not related to the general translational gas motion. All other real, flowing media are subject to the Newtonian friction law in either a definite range of the flow parameters (for a number of media, in all cases of practical interest), or always manifest a dependence of the viscosity on the flow and offer resistance to not only the tangential but also the normal stresses. The whole set of such real media are usually called non-Newtonian fluids. But namely the diversity of the properties and the multiplicity of such media of most diverse chemical nature urgently demand some sufficiently simple methods of description, while nevertheless retaining the basic physical meaning of the phenomenon (including passage to the limit to a Newtonian medium).

An exponential relationship between the flow and the friction is apparently sufficiently flexible and exact for the description of the quite numerous class of non-Newtonian media. We called a corresponding class of media which did not manifest elastic properties, structurally viscous. Its hydrodynamic and thermokinetic dependences are considered concisely below. Later the possibility is shown of extending this law to viscoelastic media as well.

The exponential rheological relationship for structurally viscous fluids which have no noticeable elastic properties is

$$\varphi^* = \exp(-\tau^*), \tag{1}$$

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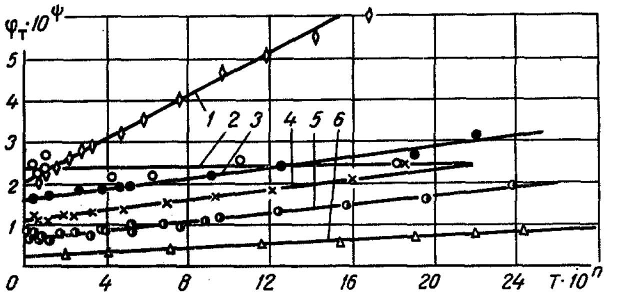


Fig. 1

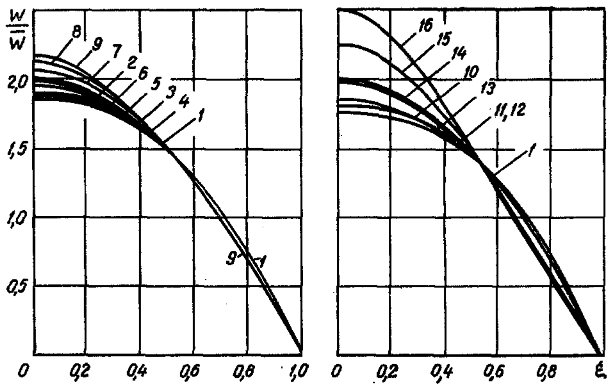


Fig. 2

Fig. 1. The dependence $\varphi_T(T)$ for a number of viscoelastic fluids: 1) linear polyethylene melt, t = 190°C, ψ = 5, n = −4 [2]; 2) Knee Joint (synovia of the knee joint), ψ = 0, n = −2 [3]; 3) polyethylene melt prior to the beginning of rupture, t = 190°C, ψ = 4, n = −5 [2]; 4) 15% solution of polyisobutylene in decalin, t = 50°C, ψ = 3, n = −2 [4]; 5) the same, t = 30°C, ψ = 3, n = −2 [4]; 6) 3% solution polyacrylamide in water, author's measurements, ψ = 0, n = −1, φ_T, m^2/N · sec; T, N/m^2.

Fig. 2. Velocity distribution for laminar flow: 1) B = 0 (Newtonian fluid); for B = 0.6: 2) A = 0.1; 3) 0.5; 4) 1; 5) 3.3; for B = −1.5: 6) A = 0.1; 7) 0.5; 8) 1; 9) 3.3; for B = 0.9: 10) A = 0.1; 11) 0.5; 12) 1; 13) 2; for B = −10: 14) A = 0.1; 15) 1; 16) 2.

where

$$\varphi^* = \frac{\varphi - \varphi_0}{\varphi_\infty - \varphi_0}, \quad \tau^* = \theta \frac{|\tau - \tau_1|}{\varphi_\infty - \varphi_0}.$$

For small values of τ^*, the flow dependence on the shear stress can be approximated by a linear law

$$\varphi = \varphi_0 + \theta|\tau - \tau_1|. \tag{2}$$

If the liquid possesses noticeable elastic properties and the shear velocity is such that the difference between the normal stresses exceeds the tangential, the character of the dependence $\varphi(\tau)$ changes. The flow increases rapidly as τ grows, and the $\varphi(\tau)$ curves are characterized by a noticeable concavity relative to the vertical axis.

It can be assumed that an anisotropy in the normal stresses, which occurs as the shear velocity increases, exerts influence on the magnitude of the flow of viscoelastic fluids. Since the majority of tests shows that the normal stresses acting in a plane perpendicular to the flow direction P_{22} and P_{33} are equal or nearly so in magnitude, the degree of anisotropy in the normal stresses can be characterized by the quantity

$$P_{11} - P_{22} \approx P_{11} - P_{33}.$$

As the processing of experimental results shows [2-4], the change in the flow of viscoelastic fluids can be described by an equation of the type (2) if the quantity

$$T = \tau + (P_{11} - P_{22}), \tag{3}$$

i.e., the relationship

$$\varphi_T = \frac{\dot{W}}{T} = \varphi_0 + \theta_T T \tag{4}$$

is used in place of τ.

To illustrate this situation, flow curves of a number of fluids possessing high elasticity are represented in Fig. 1 in $\varphi_T(T)$ coordinates.

The velocity profiles in channels of simplest shape, the hydraulic drag and heat emission coefficients for fluids possessing the linear flow law (2) have been computed earlier [1, 7].

Such fluids can evidently be considered as a linear subclass of the more general class of media with an exponential flow law.

2. Heat Exchange and Hydraulic Drag for a Stabilized Laminar Flow of Fluids with an Exponential Flow Law

Let us consider fluid flow in a circular tube. Integration of the equation $dW/dr = -\varphi\tau_W\xi$ under the condition $\tau_1 = 0$ results in the following velocity distribution law over the tube cross section:

$$\omega = \frac{W(\xi)}{\overline{W}} = \frac{0.5(1-\xi^2) + \frac{B}{A}\left[\exp(-A)\left(1+\frac{1}{A}\right) - \exp(-\xi A)\left(\xi+\frac{1}{A}\right)\right]}{0.25-6\frac{B}{A^4}+2\frac{B}{A}\left[\exp(-A)\left(0.5+\frac{1,5}{A}+\frac{3}{A^2}+\frac{3}{A^3}\right)\right]}, \tag{5}$$

where $\overline{W} = 2\int_0^1 W(\xi)\xi d\xi$ is the mean stream velocity,

$$A = \frac{\theta\tau_W}{\varphi_\infty - \varphi_0} \text{ and } B = \frac{\varphi_\infty - \varphi_0}{\varphi_\infty}.$$

The results of computing the velocity profiles for pseudoplastic ($B > 0$) and dilatant ($B < 0$) fluids are represented in Fig. 2. For pseudoplastic fluids the velocity profile is compressed for $A < 1$, while for $A > 1$ part of the section abutting the wall is occupied by a fluid with practically constant viscosity ("the second Newtonian viscosity"), and the profile again starts to approximate the parabolic. For dilatant fluids compression of the profile starts at rather higher values of the parameter A.

The dimensionless heat emission coefficient on the stabilized heat exchange section is determined under the condition of a uniform heat supply ($\partial t/\partial x \approx$ const) by the relationship [5, 6]

$$\mathrm{Nu} = \left[2\int_0^1 \frac{\left(\int_0^\xi \omega\xi\, d\xi\right)^2}{\xi}\, d\xi\right]^{-1}. \tag{6}$$

The results of computations performed using an electronic computer are represented in Fig. 3. The heat emission coefficients for pseudoplastic fluids are somewhat higher than for Newtonian fluids, and lower for dilatant fluids. There are extremal values on the curves of the dependence Nu(A) which correspond to those values of A for which the fluid flow practically ceases to vary. However, the difference between the heat emission coefficients of ordinary and non-Newtonian fluids is quite insignificant in the whole range of parameter variation. Using the relationship (5), the hydraulic drag coefficient

$$\zeta = \frac{8\tau_W}{\rho\overline{W}^2}$$

can be represented as

$$\zeta\,\mathrm{Re}_0 = \frac{16(1-B)}{0.25-6\frac{B}{A^4}+2\frac{B}{A}\left[\exp(-A)\left(0.5+\frac{1.5}{A}+\frac{3}{A^2}+\frac{3}{A^3}\right)\right]}, \tag{7}$$

where $\mathrm{Re}_0 = \varphi_0\rho\overline{W}\cdot 2\cdot R_0$. If the parameter A is transformed into $A = \beta\zeta/8$, where $\beta = [\theta/(\varphi_\infty - \varphi_0)]\rho\overline{W}^2$, then (7) is rewritten as follows:

$$\frac{16(1-B)}{\zeta\,\mathrm{Re}_0} - \left\{0.25 - 6B\left(\frac{8}{\beta\zeta}\right)^4 + 2\frac{8}{\beta\zeta}B\exp\left(-\frac{\beta\zeta}{8}\right)\left[0.5+1.5\frac{8}{\beta\zeta}+3\left(\frac{8}{\beta\zeta}\right)^2+3\left(\frac{8}{\beta\zeta}\right)^3\right]\right\} = 0, \tag{8}$$

and it can be solved for ζ. The results of a computation obtained by using an electronic computer are presented in Fig. 4. It is seen from this graph that the non-Newtonian properties of fluids alter the hydraulic drag coefficient essentially for small Re_0 numbers, while the influence of variable viscosity practically ceases to be felt as $\mathrm{Re}_0 \to \mathrm{Re}_{cr}$.

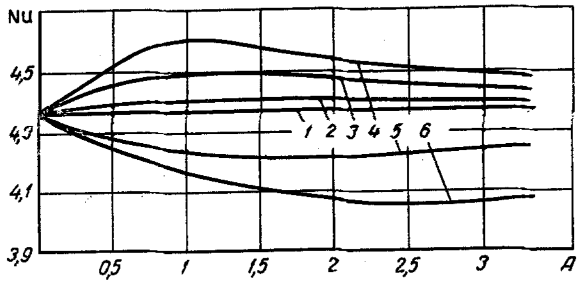


Fig. 3

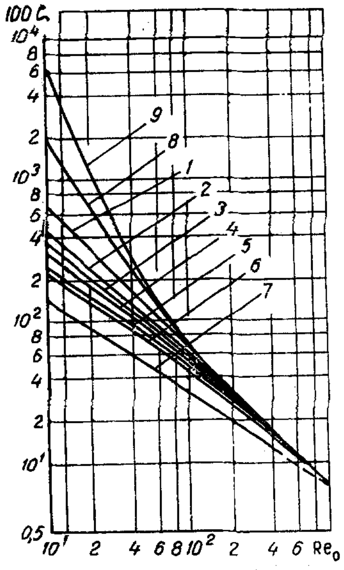


Fig. 4

Fig. 3. Heat emission coefficient for laminar flow: 1) Nu = 4.36; 2) B = 0.2; 3) 0.6; 4) 0.9; 5) −1.5; 6) −4.

Fig. 4. Hydraulic drag coefficient: 1) B = 0 (Newtonian fluid); 2) $\beta = 1$, B = 0.6; 3) $\beta = 2$, B = 0.6; 4) $\beta = 1$, B = 0.8; 5) $\beta = 2$, B = 0.8; 6) $\beta = 1$, B = 0.9; 7) $\beta = 4$, B = 0.9; 8) $\beta = 1$, B = −4; 9) $\beta = 1$, B = −10.

For the values $\beta = 1$-4, the hydraulic drag coefficient at B = 0.5-0.9 can be computed by means of the interpolation formula

$$\lg(100\zeta) = y_0 - \frac{y_0 - 0.806}{2}(\lg \mathrm{Re}_0 - 1), \tag{9}$$

where

$$y_0 = 2.806 - \frac{0.95\beta}{0.7+\beta} B^2.$$

The error in the computation will hence not exceed 1-2%.

3. Steady Laminar Flow of Viscoelastic Fluids

We consider a flow steady at such a distance from the channel entrance that the stress state of the fluid and the velocity profile can be considered completely stabilized.

If the relation between the normal and tangential stresses can be expressed by simple dependences in the shear velocity range under consideration, then the velocity profiles are easily determined by integrating (4).

For the linear viscoelasticity domain characterized by the relationship

$$\sigma/\tau = \gamma_e,$$

the integration of (4) results in the following expression for the velocity profile in a circular tube:

$$\omega = \frac{W}{\overline{W}} = 2\,\frac{1-\xi^2+\frac{2}{3}\frac{\theta_T\tau_W}{\varphi_0}(1+\gamma_e)(1-\xi^3)}{1+\frac{4}{5}\frac{\theta_T\tau_W}{\varphi_0}(1+\gamma_e)}. \tag{10}$$

This expression differs from that obtained earlier for structurally viscous fluids [1] in that the complex $\theta\tau_W/\varphi_0$ is here replaced by the quantity $(\theta_T\tau_W/\varphi_0)(1+\gamma_e)$.

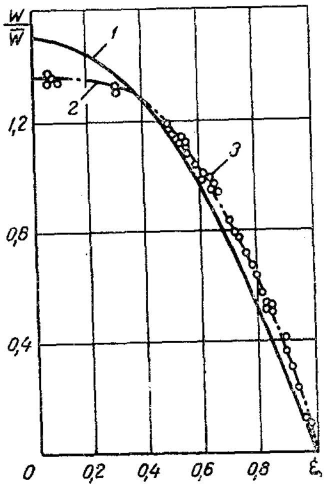


Fig. 5

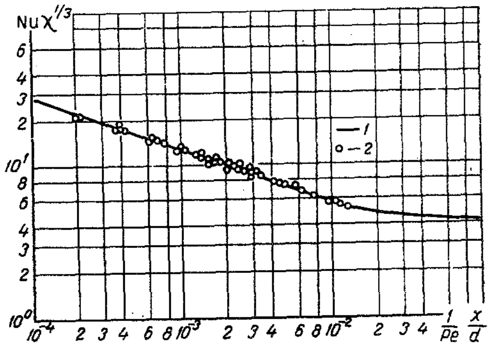


Fig. 6

Fig. 5. Velocity profiles for a 3% polyacrylamide solution in water: 1) velocity profile for Newtonian fluids; 2) computed velocity profile for a 3% polyacrylamide solution in water for $(\theta_T/\varphi_0)(1+\gamma_e) = 2.3$; 3) test values of the velocity for a 3% polyacrylamide solution in water.

Fig. 6 Test results on the heat exchange for a 3% solution of polyacrylamide in water: 1) computational curve for a Newtonian fluid; 2) test results on heat emission to the stream of a 3% polyacrylamide solution in water.

Correspondingly, the expression

$$\zeta = \frac{8\tau_w}{\rho \overline{W}^2} = \frac{5}{\beta_\tau}\left[\left(1+\frac{128\beta_\tau}{5\mathrm{Re}_0}\right)^{0.5} - 1\right]\frac{1}{1+\gamma_e}, \tag{11}$$

where

$$\mathrm{Re}_0 = \varphi_0 D\rho\overline{W} \text{ and } \beta_\tau = \frac{\theta_\tau}{\varphi_0}\rho\overline{W}^2,$$

will be valid for the hydraulic drag coefficient.

Integrating (4) results also in comparatively simple expressions for the cases when the relationship between the normal and tangential stresses is more complex, for example:

$$\sigma/\tau = A + B\tau \tag{12}$$

or

$$\tau^* = \exp\left(-\frac{\sigma^*}{\gamma_e}\right). \tag{13}$$

In practice, however, cases are quite often encountered when the quantitative characteristics of the elastic properties of the fluid are not known, and only its flow curve has been measured. A power-law dependence of the type $\tau = k\dot{W}^n$, which is physically explicitly incorrect [1], is most often used to approximate the flow curve.

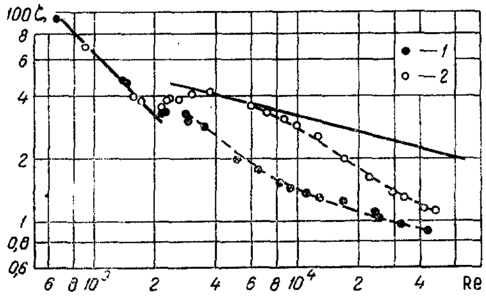
Interpolation formulas of the type

$$\varphi = \varphi_0 + \sum_{n=1}^{m} \theta_n \tau^n \tag{14}$$

not only satisfy the limit passage conditions sufficiently correctly from the physical and mathematical viewpoints, but are also sufficiently convenient from the purely computational viewpoint.

It should be noted that the velocity profiles are made flatter as the elastic properties of the fluid grow (the quantity γ_e), and the velocity gradient at the wall increases.

Using the relationship (14), the heat emission coefficient of non-Newtonian fluids for the warm initial section of a tube can be determined by means of the formula [7]



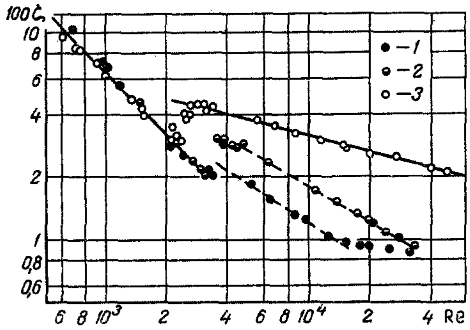


Fig. 7 Fig. 8

Fig. 7. Hydraulic drag coefficient in a tube for a 0.02% polyethylene oxide solution: 1) fresh solution; 2) solution subjected to destruction.

Fig. 8. Hydraulic drag coefficient in a tube for a 0.015% polyacrylamide solution: 1) fresh solution; 2) solution subjected to destruction; 3) water.

$$\mathrm{Nu} = \mathrm{Nu}_0\,\chi^{1/3}, \tag{15}$$

where

$$\chi = \left(1 + \sum_{n=1}^{m} \frac{\theta_n}{\varphi_0}\,\tau_w^n\right)\left(1 + \sum_{n=1}^{m} \frac{4}{n+4}\,\frac{\theta_n}{\varphi_0}\,\tau_w^n\right)^{-1}. \tag{16}$$

As the tangential shear stress grows, the quantity χ increases, however it exceeds 1 insignificantly. Thus, for pseudoplastic fluids with a linear flow law $\chi \to 1.25$ as $\tau \to \infty$, while $\chi \to 1.5$ in the domain of the quadratic flow law. Correspondingly, the ultimate increase in the heat emission coefficients is just 8 and 15% as compared with their values for Newtonian fluids.

4. Measurement of the Velocities, Drag Coefficients, and Heat Emission Coefficients in Non-Newtonian Fluid Streams

Measurement of the velocity profiles in non-Newtonian fluid streams is complicated by a number of factors. The fluid viscoelastic properties affect the operation of the total head tube, and it is not clear at present how to interpret the readings obtained by using them in either the laminar or turbulent flow regions. Attempts to use a thermoanemometer to investigate turbulent flow of weakly concentrated polymer solutions have also been unsuccessful.

We used optical flow-visualization methods to investigate the velocity fields in structurally viscous and viscoelastic fluid streams. In the first series of tests the fluid velocities were measured by an optico-mechanical device [8], and in subsequent experiments by using an electronic stroboscope for hydrodynamic investigations.

One of the measured velocity profiles is represented in Fig. 5 for the laminar flow of a 3% polyacrylamide solution possessing considerable elastic properties. Measurements of these properties by using an instrument of cone–plane type showed that the difference between the normal stresses exceeded the tangential stresses at the wall approximately one- to twofold at the shear velocities holding in the experiments. The experimental apparatus used in this series of tests was two tanks connected by a flat channel with 1 : 10 side ratio. The tangential stresses at the channel wall varied between 10 and 110 N/m^2, which corresponded to flow velocities between $7.4 \cdot 10^{-4}$ and $7.1 \cdot 10^{-2}$ m/sec. The velocity profiles obtained agreed with the computed ones, and in contrast to [9, 10], no slip was noted near the wall in the tests.

The results of heat-exchange tests in the laminar flow mode are presented in Fig. 6. The heat emission coefficients of a 1% polyacrylamide solution were measured in a circular tube of 1 cm diameter and 90 cm length under the condition of uniform heat supply to the tube surface. The prepared polyacrylamide solution was poured into a large tank from which it was pumped through the working section under air pressure. The fluid discharge was measured by a volume method. The fluid temperature at the entrance to the working section, the temperature along the length of the heated tube, and the temperature drop in the heat insulation in the working section were all measured. Fluid heating in the working section is determined by computations taking account of the heat losses of the heater.

The tests were performed for different values of the heat flux so that the heat emission coefficients could be determined under quasiisothermal conditions by extrapolation to zero heat flux. The results of tests for quasiisothermal conditions are in good agreement with the computational relationship (15). We noted a similar agreement earlier in an investigation of the heat emission to carboxymethylcellulose solutions flowing in a rectangular channel [11]. It should be noted that the influence of the non-Newtonian fluid properties on the heat emission coefficient is weaker than the change in viscosity over the channel cross section because of the heat supply.

An electronic stroboscope was used for hydrodynamic investigations [12, 13] to obtain velocity profiles for turbulent non-Newtonian fluid flow. Weakly concentrated high-polymer solutions were selected for the investigations: carboxymethylcellulose (CMC), desoxyribonucleic acid (DNA), polyacrylamide (PAM), and polyethylene oxide.

It has been shown in [14] that the introduction of polymer admixtures exerts the greatest influence on the intermediate stream domain. Subsequent measurements with polyethylene oxide solutions confirmed the conclusions made earlier. An increase in the size of the intermediate domain, of the magnitude of the velocity on its boundary with the domain of completely developed turbulent flow, should result in a reduction in the hydraulic drag and heat emission coefficients.

The results of measuring the hydraulic drag during circulation of polyethylene oxide solutions (0.02%) in a closed loop are represented in Fig. 7. The tests were conducted in a d = 8 mm circular tube. The lower line on the graph corresponds to the first series of tests with fresh polyethylene oxide solution. The fluid discharge in this series of tests was gradually increased from values corresponding to the laminar flow mode to the highest possible. Rapid destruction of the solution started at the high flow speeds, and as the discharge was reduced the hydraulic drag coefficients were ranged along the upper branch of the curve, where addition of a polymer for low Reynolds numbers in the turbulent flow domain already has practically no effect on the hydraulic drag of the stream.

Presented in Fig. 8 are results of measuring the hydraulic drag in polyacrylamide solutions of 0.015% concentration. The data obtained differ somewhat from the preceding in that the transition from the laminar to the turbulent flow mode is rather stretched out, the destruction of the solution is less, and the introduction of an admixture is felt in the turbulent mode even at low Re numbers.

The results of investigating the pulsating velocity profiles over the whole channel cross section (including even the viscous sublayer) were described partially in [14] and were discussed in detail in a paper published in this issue (p. 758).

A series of tests was conducted to determine the heat emission coefficients in a turbulent flow of polyethylene oxide and polyacrylamide solutions. The tests showed that the coefficients of heat emission to a stream of solution can be severalfold lower than to a stream of water for the same values of the Re and Pr numbers. The reason for this is the increase in thermal resistance of the intermediate stream domain.

NOTATION

$\tau = \tau_W \xi$	is the tangential shear stress, N/m^2;
τ_W	is the tangential shear stress at the wall, N/m^2;
φ_0	is the flow at $\tau \to 0$, $m^2/N \cdot sec$;
φ_∞	is the flow at $\tau \to \infty$, $m^2/N \cdot sec$;
θ	is the coefficient of instability of the structure, $m^4/N^2 \cdot sec$;
τ_1	is the limit shear stress at which non-Newtonian properties start to appear in the fluid, N/m^2;
$\sigma_1 = P_{11} - P_{22}$, $\sigma_2 = P_{11} - P_{33}$	are the first and second difference in the normal stresses, respectively, N/m^2;
$\dot{W}$	is the velocity gradient, sec^{-1};
R	is the tube radius, m;
$\xi = r/R$	is the dimensionless distance from the wall;

Re_0	is the Reynolds number determined at zero flow;
Re_{cr}	is the critical Reynolds number;
γ_e	is the magnitude of highly elastic deformation;
χ	is the coefficient taking account of the structural viscous properties of the medium;
Nu_0	is the heat emission coefficient for Newtonian fluids;
Pr	is the Prandtl number;
ξ	is the hydraulic drag coefficient;
$\tau^* = (\tau_{cr} - \tau)/\tau_{cr}$;	
$\sigma^* = \sigma/\tau_{cr}$;	
θ_T	is the instability coefficient of the elastic structure;
τ_{cr}	is the critical value of the tangential shear stress.

LITERATURE CITED

1. S. S. Kutateladze, V. I. Popov, and E. M. Khabakhpasheva, Prikl. Mekh. i Tekh. Fiz., No. 1, 49 (1966).
2. E. B. Bagley and H. P. Shreiber, Trans. of the Soc. of Rheology, 5, 341 (1961).
3. R. G. King, Rheologica Acta, 5, Heft 1, 35 (1966).
4. W. Philippoff, J. Appl. Phys., 27, No. 9, 984 (1956).
5. R. Lyon, Chem. Eng. Progress, 47, 75, II (1951).
6. S. S. Kutateladze, Principles of the Theory of Heat Exchange [in Russian], Mashgiz (1962).
7. V. I. Popov and E. M. Khabakhpasheva, Prikl. Mekh. i Tekh. Fiz., No. 3, 146 (1966).
8. Yu. V. Kostylev, V. I. Popov, and E. M. Kabakhpasheva, Prikl. Mekh. i Tekh. Fiz., No. 2, 100 (1966).
9. I. Galt and B. Maxwell, Modern Plastics, No. 4, 42, 115 (1964).
10. Yu. A. Zhdanov and V. F. Dubovitskii, Heat and Mass Transfer [in Russian], No. 3, Nauka i Tekhnika, Minsk, p. 241.
11. V. I. Popov and E. M. Khabakhpasheva, Inzh.-Fiz. Zh., 12, No. 2, 278 (1967).
12. V. V. Orlov and Yu. M. Fedoruk, Teplofiz. Vysok. Temper., No. 4, 764 (1968).
13. E. M. Khabakhpasheva, B. V. Perepelitsa, E. S. Mikhailova, V. V. Orlov, V. M. Karsten, and G. I. Efimenko, Heat and Mass Transfer [in Russian], No. 1, Énergiya, Moscow (1968), p. 165.
14. E. M. Khabakhpasheva and B. V. Perepelitsa, Inzh.-Fiz. Zh., 14, No. 4, 598 (1968).